6. Boolean Algebra Isomorphism:-

Let B_1 & B_2 be two Boolean algebras.

$f: B_1 \rightarrow B_2$ is called B.A.I

if i) f is 1-1

ii) f is onto

iii) $f(a+b) = f(a) + f(b)$

$f(ab) = f(a) \cdot f(b)$

iv) $f(a') = [f(a)]' \; \forall a \in B$ $\quad \therefore B_1 \cong B_2$

Thm B be finite Boolean Algebra & A be set of Atoms of B. If $P(A)$ is B.A. of all subsets of set A.

$$f : B \to P(A)$$

$$f(x) = \{a_1\ a_2 \ldots a_n\}$$ is isomorp.

$x = a_1 + a_2 + \ldots + a_n$ is unique repres. of x as sum of atoms

(~~Representation Theorem~~)

Proof :- Let A be Atom,

$$A = \{ a \in A,\ a \le x \}$$

$$f : B \to P(A)$$

$$f(x) = A(x)$$

$$= \{a_1, a_2 \ldots a_n\}$$

$$b_1\ b_2 \ldots b_s$$

T.P $B \cong P(A)$.

i) Homomorphism :-

Let $x, y \in B$

$$x = a_1 + a_2 + \ldots + a_r + b_1 + b_2 + \ldots + b_s$$

$$y = b_1 + b_2 + \ldots + b_s + c_1 + c_2 + \ldots + c_t$$

$$A = \{ a_1, a_2, \ldots a_r, b_1, b_2 \ldots b_s, c_1\, c_2 \ldots c_t, d_1\, d_2 \ldots d_k \}$$

$$x + y = (a_1 + a_2 + \ldots + a_r + b_1 + b_2 + \ldots b_s) \cup (b_1 + b_2 + \ldots + b_s + c_1 + c_2 + \ldots + c_t)$$

$$x + y = a_1 + a_2 + \ldots + a_r + b_1 + b_2 + \ldots + b_s + c_1 + c_2 + \ldots c_t$$

$$\& \quad xy = (a_1 + a_2 + \ldots + a_r + b_1 + b_2 + \ldots b_s) \cap (b_1 + b_2 + \ldots + b_s + c_1 + c_2 + \ldots c_t)$$

$$= b_1 + b_2 + \ldots + b_s.$$

$f(x+y) = f(a_1+a_2+\cdots+a_r+b_1+b_2+\cdots+b_s+c_1+c_2+\cdots c_t)$

$= \{a_1, a_2, \ldots a_r, b_1, b_2 \ldots, b_s, c_1, c_2, \ldots c_t\}$

$= \{a_1, a_2 \ldots a_r, b_1, b_2 \ldots b_s\} \cup \{b_1 b_2 \ldots b_s, c_1 c_2 \ldots c_t\}$

$= f(x) \cup f(y)$

& $f(xy) = f(b_1+b_2+\cdots+b_s)$

$= \{a_1, a_2 \ldots a_r, b_1, b_2 \ldots b_s\} \cap \{b_1, b_2 \ldots b_s, c_1, c_2 \ldots c_t\}$

$= f(x) \cap f(y)$

ii) ~~Let $y = c_1+c_2+\cdots+c_t+d_1+d_2+\cdots+d_k$~~

~~$f((xy))$~~ Let $y = c_1+c_2+\cdots+c_t+d_1+d_2+\cdots+d_k$

$x = a_1+a_2+\cdots a_r+b_1+b_2+\cdots+b_s$

$x' = y$

$f(x') = f(y)$

$= \{c_1, c_2 \ldots c_t, d_1, d_2 \ldots d_k\}$

$= \{a_1 a_2 \ldots a_r, b_1 b_2 \ldots b_s\}'$

$= [f(x)]'$

iii) Every Element of B can be uniquely expressed as sum of finite No. of atoms.

The representation is unique

$\therefore$ f is one-one & onto.

Hence f is Boolean Algebra isomorphism.

$B \cong P(A)$.